velocity is reached, lift L and thus C_L must vanish, in accordance with Eq. (4). In this special case, Eq. (13) reduces to

$$R_s{}^{-1} = B_0 \tag{20}$$

and Eqs. (15) is replaced by

$$R = -\left(\frac{1}{B_0}\right)\int_{W_i}^{W_f} \mathrm{d}W \tag{21}$$

which can be integrated to give

$$R = (W_i - W_f)/B_0 \tag{22}$$

Also,

$$E = R/V_r = (W_i - W_f)/B_0 V_r \tag{23}$$

Application to Vehicle Optimization

To study the effect of hypersonic Mach number on the specific range, it is convenient to replace C_{D_0} and K as specified in Eq. (6). Then, for $V^2/gR_0 < 1$

$$R_s^{-1} = a_{-2}M^{-2} + a_{-1}M^{-1} + a_0 + a_1 M + \tfrac{1}{4}M^2 + a_3 M^3 \tag{24}$$

where the a are extensive algebraic expressions of the parameters.

At orbital speed $V^2/gR_0 = 1$ and

$$R_s^{-1} = B_0 = \tfrac{1}{2} C_{D_1} S\rho V_a (C_0 + C_l M + cC_1 M^{-2})$$

or

$$R_s^{-1} = a'_{-2}M^{-2} + a'_0 + a'_1 M \tag{25}$$

where

$$a'_{-2} = \tfrac{1}{2} C_{D_1} C_1 c S\rho V_a, \quad a'_0 = \tfrac{1}{2} C_{D_1} C_0 S\rho V_a$$

$$a'_1 = \tfrac{1}{2} C_{D_1} C_1 S\rho V_a \tag{26}$$

Setting $\mathrm{d}(R_s^{-1})/\mathrm{d}M = 0$ yields

$$M_{\mathrm{opt}} = (2a'_{-2}/a'_1)^{1/3} = (2c)^{1/3} = M_m \tag{27}$$

$$(R_s)_{\mathrm{opt}} = a'_{-2}M_m^{-2} + a'_0 M_m \tag{28}$$

Equation (27) shows that the optimal Mach number to achieve maximum range in orbit is M_m, the Mach number at which the $-\dot{W}/T$ is a minimum.

Acknowledgments

Helpful discussions with colleagues, Profs. Karl H. Bergey, George Emanuel, and Maurice L. Rasmussen, are gratefully acknowledged.

References

[1] Williams, R.M., "National Aero-Space Plane: Technology for America's Future," *Aerospace America*, Vol. 24, Nov. 1986, pp. 18–22.

[2] Etkin, B., "Longitudinal Dynamics of a Lifting Vehicle in Orbital Flight," *Journal of the Aerospace Sciences*, Vol. 28, Oct. 1961, pp. 778–779, 832.

[3] Etkin, B., *Dynamics of Atmospheric Flight*, Wiley, New York, 1972, pp. 146–147.

[4] Bushnell, D.M., "Overview: Technology Issues," Presentation at State University of New York at Buffalo/Calspan Research Center Short Course on Hypersonics, Buffalo, Aug. 1986.

[5] Drummond, A.M., "Performance and Stability of Hypervelocity Aircraft Flying on a Minor Circle," *Progress in Aerospace Sciences*, Vol. 13, edited by D. Küchemann, Pergamon Press, Oxford, England, 1972, pp. 137–221.

[6] Bert, C.W., "Prediction of Range and Endurance of Jet Aircraft at Constant Altitude," *Journal of Aircraft*, Vol. 18, Oct. 1981, pp. 890–892.

[7] Hoerner, S.F., "Fluid-Dynamic Drag," Hoerner Fluid Dynamics, Brick Town, NJ, 1965, pp. 17–18.

[8] Wood, K.D., "Aerospace Vehicle Design, Aircraft Design," Vol. I, Johnson Publishing, Boulder, CO, 1968, p. 44.

[9] *AIAA Aerospace Design Engineers Guide*, American Institute of Aeronautics and Astronautics, Washington, DC, 1983, pp. 7-4 and 8-3.

[10] Ferri, A., "Supersonic Combustion Progress," *Astronautics and Aeronautics*, Vol. 2, Aug. 1964, pp. 32–37.

[11] Mordell, D.L. and Swithenbank, J., "Hypersonic Ramjets," *Advances in Aeronautical Sciences*, Vol. 4 (Proceedings of 2nd International Congress in the Aeronautical Sciences, Sept. 1960), Pergamon, New York, 1962, pp. 831–847.

Optimum Structural Sizing for Gust-Induced Response

P. Hajela* and C. T. Bach†
University of Florida, Gainesville, Florida

Introduction

THE use of mathematical nonlinear programming algorithms has enjoyed considerable success in the automated structural synthesis environment. A majority of research pertaining to optimum structural design has focused primarily on statically loaded structures, and more effort needs to be directed at developing sizing capabilities for dynamic loads, in particular, nondeterministic loads.[1] The optimum sizing of airfrace structures requires an analysis tool that accounts for static and dynamic structural stability, deformations under applied loads, and the interaction of structural deformations and airloads.

The present work was directed toward establishing an optimization capability for sizing wing structures that are subjected to a combination of deterministic and random flight loads. This included the implementation of efficient methods for computing response sensitivity required by the optimization algorithm. The random loads were treated as a stationary, homogeneous process with a Gaussian distribution. A frequency-domain analysis was selected for the solution of the dynamic response problem, wherein the gust loads were represented by a power spectral density spectrum of gust velocities.

For a structure subjected to nondeterministic loads, failure can result either from a single exceedance of stress, or from cumulative damage due to fatigue. For these failure modes, Johnson[2] formulates design constraints applicable only when a single stress component is considered critical in the constraint definitions. Constraints for fatigue failure have also been obtained from a fracture mechanics standpoint.[3] If a combination of the response quantities is involved, as in the

Received March 4, 1988; presented as Paper 88-2298 at the AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, Williamsburg, VA, April 18–20, 1988; revision received June 15, 1988.

*Associate Professor, Aerospace Engineering, Mechanics, and Engineering Science. Member AIAA.

†Graduate Research Assistant, Aerospace Engineering, Mechanics, and Engineering Science. Student Member AIAA.

von Mises failure criterion, the phase information necessary to combine the response components is difficult to obtain. An important contribution of this work is in the development and implementation of sizing constraints based on an equal-probability-of-load-combination criterion.[4]

Structural Reliability Constraints

For a response $s(t)$ that is stationary and has a Gaussian distribution, an assumption that large values of $s(t)$ arrive independently of one another, leads to a Poisson probability function for the number of times that a large magnitude S_s is exceeded in time t. If the occurrence of this large stress magnitude is not permitted in time L_s, the single excursion (SE) constraint can be written as follows:

$$g_{\mathrm{SE}} = 1 - L_s\left[\frac{\sigma_{\dot{s}}}{\sigma_s \pi}\exp\left(-\frac{S_s^2}{2\sigma_s^2}\right)\right] \geq 0 \tag{1}$$

Fatigue damage can be estimated on the basis of the Palmgren-Miner theory and results in a fatigue constraint as follows:

$$g_F = 1 - L_F\left[\frac{\sigma_{\dot{s}}}{2\pi\sigma_s c}(2\sigma_s^2)^{b/2}\,\Gamma\left(\frac{b+2}{2}\right)\right] \geq 0 \tag{2}$$

where L_F is the specified fatigue life, Γ is the gamma function, and b and c are constants obtained from empirical relations for different materials. In Eqs. (1) and (2), σ_s and $\sigma_{\dot{s}}$ are the rms response and rms response rate, respectively.

In the preceding analysis, if $s(t)$ were a function of two or more response components, determination of the rms and rms rate of $s(t)$ becomes more difficult, particularly if it is a nonlinear function.[1] If the rms quantities for the individual components were to be obtained, and this is a relatively simple task, their utility would be restricted by the lack of phase information necessary to combine them. Note that rms values have only a magnitude but no direction associated with them. An example of this is obtained if one considers a stress function that is a combination of the principal stresses σ_1 and σ_2.

$$s(t) = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2} \tag{3}$$

In the absence of phase information necessary to combine the stress components, a worst-case estimate of Eq. (3) is written as follows:

$$s(t) = \sqrt{\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2} \tag{4}$$

From the standpoint of obtaining lightweight structures, this strategy is frequently undesirable, and a more meaningful approach is obtained if one uses an equal-probability-of-load-combination criterion to combine the components. For two response components x and y, we have the familiar bell-shaped normalized probability density function. Constant values of probability density $p(x,y)$ yield elliptical contours parallel to the x-y plane.

$$\left(\frac{x-\mu_x}{\alpha\sigma_x}\right)^2 + \left(\frac{y-\mu_y}{\alpha\sigma_y}\right)^2 - \frac{2\rho_{xy}}{\alpha^2}\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) = 1 \tag{5}$$

Here, α is a constant; μ_x, μ_y and σ_x, σ_y are the mean and rms values of the response quantities, respectively, and ρ_{xy} is the correlation coefficient. These quantities define the geometry and orientation of the ellipse. For a combination involving three response components, an ellipsoid is obtained for the constant probability surface.

Stauffer and Hoblitt[5] obtain a finite number of load combinations from the constant probability ellipse by circumscribing it with an octagon and using the eight vertices of the octagon as critical load combinations in the design process. This technique loses value when a larger number of load combinations for which the constraints are evaluated.

Problem Statement

A mathematical statement of the optimization problem can be written as follows:

$$\text{Minimize } F(d) \tag{6}$$

subject to the inequality constraints

$$g_j(d) \leq 0, \qquad j = 1,2,\ldots m \tag{7}$$

and side constraints on the design variables,

$$d_i^l \leq d_i \leq d_i^u \tag{8}$$

Here, $F(d)$ represents the structural weight; $g_j(d)$ are failure constraints obtained from the single exceedance or fatigue failure criterion; and d_i^l and d_i^u are the lower and upper bounds on the structural member dimensions d_i, respectively. The large number of inequality constraints obtained in the process of discretizing the constant probability surface were represented by a single cumulative constraint function as follows:

$$G = g_{\max} + (1/\rho)\ln\{\Sigma \exp[\rho(g_i - g_{\max})]\} \tag{9}$$

Here, $g_{\max}$ is the most critical constraint of the set g_i. A feasible usable search direction approach was used to obtain a solution of the preceding constrained optimization problem.

Optimization System

The optimization system developed in the present task was a modular collection of analysis and optimization programs coupled by pre- and postprocessors. The system consists of a finite-element program called EAL (Engineering Analysis Language), a nonlinear programming-based optimization program ADS (Automated Design Synthesis), and a system of aeroelastic response analysis programs ISAC (Interaction of Structure with Unsteady Aerodynamics and Control). The EAL program was used to obtain the mode shapes, eigenvalues, generalized masses, stress, displacement, and acceleration coefficients. It was also used to generate the gradients of the eigenvalues and eigenmodes by both the finite-difference and semi-analytical methods.[6]

The ISAC system of routines was employed to compute the aeroelastic response of the flight vehicle. The structural eigenmodes obtained in EAL were related to the deflections and slopes on the aerodynamic boxes by a two-dimensional spline interpolation technique. These deflections and slopes were used in a doublet lattice program to compute the generalized aerodynamic forces for a range of reduced frequencies. These matrices, along with the generalized mass and stiffness, and a selected gust spectrum, were processed to obtain the rms values of the aeroelastic response parameters. In addition, the correlation coefficients between various response components that were needed in the equal-probability-of-load-combination formulation of the design constraints were also obtained.

Numerical Examples

A built-up finite-element model of an aluminum wing structure (Fig. 1) was developed as a test problem for the program implementation. The structure was initialized for 1-g cruise loads, and these dimensions were established as lower bounds for the gust design. The first six elastic modes were chosen to

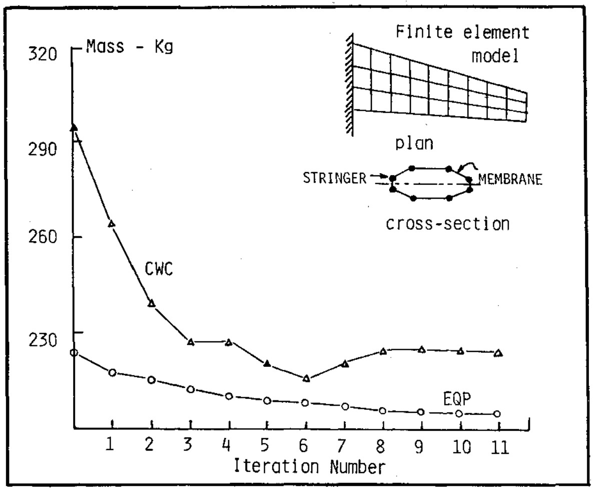

Fig. 1 Objective function convergence histories for both conserative worst-case strategy (CWC) and method, using the equal probability criterion (EQP).

represent the structural deformation for the cantilevered wing. Addition of rigid-body modes in plunge and pitch presents no additional difficulty. A 2.54 m/s (typical of storm conditions) intensity, Dryden gust spectrum, was selected for the input load.

In the first example, cross-sectional areas of bar elements were selected as the design varibles. An allowable stress of 1761.4 kg/cm^2 and values of $L_s w = L_F = 17{,}250$ h were used in the optimum design process. The first bending mode dominated the stress distribution as evidenced by a concentration of material at the root, and the first excursion constraint was active at the optimum. Both finite-difference and semi-analytical gradient computations were used with essentially similar results.

A second set of numerical results reported here pertains to the implementation and verification of constraints based on an equal-probability-of-load-combination criterion. This problem involved a combination of deterministic static loads and gust-induced random loads. The thickness of six sets of panel members was altered during redesign. A $p(x,y)$ value of 0.99 was selected for the equal probability criterion. An allowable stress magnitude of 2465.9 kg/cm^2 was specified, and a total of 40 load combinations were chosen to represent the ellipse of equal probability. Optimum designs were obtained on the basis of a worst-case estimate of the stress function given by Eq. (4) and the use of Eq. (3) with values of σ_1 and σ_2 obtained from an equal probability criterion. As shown in Fig. 1, the final optimum weight of the equal-probability-of-load-combination method was 8% less than that obtained from a conservative worst-case strategy. Additional results and a detailed description of the model are presented in Ref. 6.

Concluding Remarks

The principal goal of this study was to develop an optimization capability to design airframe structures for random gust loads in addition to static or dynamic deterministic loads. The combination of optimization algorithms with analysis tools such as EAL and ISAC makes available a tool for stress, displacement, frequency, flutter, and gust-response-constrained optimization. Furthermore, implementation of semi-analytical gradient calculations provides added computational efficiency to the programming system. This system also provides a natural test bed for continuing studies in a multilevel decomposition approach for aeroservoelastic synthesis. The use of an equal-probability-of-load-combination criterion for structural reliability constraints has also been demonstrated for representative structural models.

Acknowledgment

Research support from NASA Langley Research Center under Grant NAG 1-688 is gratefully acknowledged.

References

[1]Hajela, P., "Comments on Gust Response Constrained Optimization," NASA CP-2327, Vol. 2, April 1984.

[2]Johnson, E. H., "Optimization of Structures Undergoing Harmonic or Stochastic Excitation," SUDAAR 501, Stanford Univ., CA, Aug. 1976.

[3]Hajela, P. and Lamb, A., "Automated Structural Synthesis for Nondeterministic Loads," *Computer Methods in Applied Mechanics and Engineering,* Vol. 57, 1986, pp. 25–36.

[4]Gross, D. W. and Sobieski, J. E., "Application to Aircraft Design of Nonlinear Optimization Methods Which Include Probabilistic Constraints," AIAA Paper 80-0153, Jan. 1980.

[5]Stauffer, W. A. and Hoblitt, F. M., "Loads Determination in the Design of the L-1011," *Journal of Aircraft,* Vol. 10, Aug. 1973, pp. 459–467.

[6]Hajela, P. and Bach, C. T., "Optimum Structural Sizing for Gust Induced Response," *Proceedings of 29th AIAA/ASME/ASCE/AHS/ASC SDM Conference,* AIAA, Washington, DC, 1988.

Interactive Boundary-Layer Calculations of a Transonic Wing Flow

Kalle Kaups*
Douglas Aircraft Company, Long Beach, California

Unmeel Mehta†
NASA Ames Research Center, Moffett Field, California
and
Tuncer Cebeci‡
Douglas Aircraft Company, Long Beach, California

Introduction

THE so-called wing C was designed by NASA and the Lockheed-Georgia Company as one in a series for which it was intended to provide reliable experimental data for the purpose of comparisons with computational efforts. The full potential inviscid transonic wing code FL022, in combination with an optimization routine, was used to configure the wing for highly three-dimensional flow by selecting a large sweep angle and a low aspect ratio combined with supercritical sections and considerable twist. It was intended that the flow remain attached at a design Mach number of 0.85 and lift coefficient of 0.5, which corresponds to a 5 deg of angle of attack. The desired pressure distribution was specified at two spanwise locations and the wing was constructed by linear development between the root and tip.

The purpose of this Note is to present results obtained from interactive solutions of inviscid and boundary-layer equations and to compare them with experimental values. Calculated results were obtained with an Euler code and a transonic

Received July 6, 1988; revision received Aug. 8, 1988.

*Senior Staff Engineer, Aerodynamics Research and Technology Group. Member AIAA.

†Research Scientist. Associate Fellow AIAA.

‡Staff Director, Aerodynamics Research and Technology Group. Fellow AIAA.